Zitex Filter Requirements and Other Possibilities

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Objective
Incident light is reflected, transmitted, or absorbed within filter materials used in the cryostat window system. Of particular interest to us is light that is absorbed and re-emitted back up to the atmosphere by the IR-blocking filters. My objective was to calculate the amount of heat absorbed by Zitex filters to estimate the feasibility of using them in the QUIET cryostat to reduce IR loading. This calculation is based on the assumption that the thermal loss due to re-emittance happens at each layer and so I model the absorption at each layer and propagate through the layers to calculate total thermal loss.

1 Description of Zitex

I obtained material properties for Zitex from the polymer company St.Gobain. Zitex is made of PTFE (Teflon) with various pore sizes, depending on the grade. Typical grades are Extra Fine G110 (1-3 micron), Fine G108 (3-5 micron), Medium A145 (10-20 micron), and Coarse A135 (20-30 micron). Available thicknesses are 8-12 mil, 6-10 mil, 3.7-5.7 mil, and 4.5-6.0 mil and average pore volume of 40%, 45%, 65%, respectively. The Zitex A brands are "reproductions of standard filter papers and consquently have the same tortuous, fibrous filter paths" (as a side note, it is resistant to fungi and the spec sheet assures me it will not spontanously combust, so we can breathe easily on these counts). Zitex G is finer and and serves as a better dielectric match to PTFE [1].

2 Previous Measurements

Zitex is teflon with small bubbles of air. This leads to wavelength dependent behaviour for scattering and absorption: light with wavelengths lower than a characteristic wavelength will scatter while larger wavelengths will be absorbed in the teflon. The characteristic wavelength for scattering is theoretically found using an analogy to embedded dielectric spheres and is $\lambda = \pi a(n-1)$ where

a is the radius of the sphere, n is the index of refraction of the material. Wavelengths above this will generally be absorbed. For Zitex G115, wiht a pore size of 1-2 microns and in Teflon (n=1.44), wavelengths above 0.69 - 1.38 microns will be absorbed. The actual wavelength dependence of the absorption depends on the properties of teflon. Benford et al 2003 measured transmission of Zitex G125 (the thickest at 0.25") and found attenuation:

$$\alpha = 40exp[-\frac{\lambda}{21}^{0}.63]Np/cm \tag{1}$$

Transmission, ignoring absorption, was found to be:

$$T = exp[-145\lambda^{-1.06}] \tag{2}$$

all with lambda in microns. Upon layering Zitex, one layer directly on top of the other, they found that for the mid-IR band there was no substantial change in transmission (because scattering is the dominant effect in this range) so multiple sheets do not perform better than single sheets. By separating the layers (they used a spearation of 7mm) they were able to change the transmission slightly for longer wavelengths, indicating that absorption was beginning to contribute. Curves were given for other types of Zitex, but in wavelength ranges 1-188 microns(or frequency ranges 3 THz to 1.6 THz). This outside of our observing range of 40 and 90 GHz. They do mention that because scattering is dominant for most of the spectrum, overall absorption is low [2]. Because no measurements of multiple layers have been done in our observing bands, I calculated total absorption, and hence the temperature, on each layer of Zitex following a code used by CLarke and D'Addario in an ALMA memo. This technique is explained below, but first a few caveats:
-****They use an absorption coefficient that is constant in temperature and frequency. Zitex clearly exhibits wavelength dependent absorption, as can be seen in a subsequent ALMA memo [3]. This later memo reports results for measurements of transmission for one layer and then for multiple layers (of G110, slightly thinner Zitex), and the curves for absorption vary with frequency and temperature. For room temperature, Zitex G110 varies between -0.1 - -0.4 dB in a frequency range 75-110 GHz. The sample at 77K shows a much greater frequency dependence, varying from -0.9 dB at 82.5 GHz to -0.05 dB at 98 GHz, and then levels off. This frequency dependent transmission ing our observing band at lower temperatures is very troubling, and as a result we are considering other filter materials. This depenedence is due to interference effects as the Zitex stiffens during cooling. Quantitatively, for a similar percentage change in absorption in our Zitex (80%), the absorption coeffcient of 0.69 given in [3] could vary

from 0.13 to 1.
-They ignore reflection
- There is no way of knowing how much the spacing between the filters effected their results without measuring it ourselves.
-There is no way of doing this for other types of Zitex with different thicknesses and pore sizes without measuring it ourselves.

3 Calculations

I used the radiation calculation presented in Appendix A of [4] and extended it to more layers to calculate the number of layers necessary to block 90% of the heat coming from higher frequency photons.
We begin by studying the propagation of radiation through the layers. We will ignore reflection, assuming it is small, and define absorption as a and transmission as (1-a).
Layer 1: Energy In = P_1
Energy Absorbed: aP_1
Energy Out: (1-a)P_1
Layer 2: Energy In: (1-a)P_1
Energy Absorbed = a(1-a)P_1
Energy Out = (1-a)(1-a)P_1
We can see that a pattern will arise: the amount of energy being radiated by any layer is $(1-a)^N P_1$, and the exponent for the absorption at one layer is always 1. We can create a matrix to store this information, it will describe the fraction of power radiated by the layer j towards layer i and absorbed there. The matrix will then take the form:

$$f(i,j) = a(1-a)^{|i-j|-1} \tag{3}$$

Intuitively, the amount of flux present in layer 3 from layer 1 is simply a(1-a)*Power in layer 1. We must change the diagonal elements by hand; they represent the amount of heat radiated away from only one layer. This layer does not absorb, in fact it emits equally in two directions, and so the fraction of power absorbed and emitted is -2, (-1 would be for emission on only one side, the -2 indicates its emits equally in both directions). Now we are ready to solve for someting useful. The basic idea is that the total flux (incident+transmitted+absorbed) in each layer is zero, except for the top which is recieving radiation flux from the window, and the bottom which is absorbig 4K flux as a boundary condition. We can use the absorption matrix above to calculate this flux:

$$B = A * P \tag{4}$$

Where A is the absorption matrix above, P is a matrix that holds values for incident flux, and hence B is the total flux through the layer. Because the layers are in isolation, the net flux is always zero, except for the top and bottom layers (with radiation at 300K and 4K, respectively). We know A, the only matrix we don't know is the incident power on each layer, P.

$$P = A^{-1} * B \tag{5}$$

We also note that $Power = \sigma\epsilon T^4$. The B vector is populated by $\sigma 300^4$ and $\sigma 4^4$. From equating the two power equations, we can see that σ will cancel but a factor of ϵ, the emissivity, will remain. We will divide through by ϵ, so our B vector only had the two non-zero components $\frac{300^4}{\epsilon}$ and $\frac{4^4}{\epsilon}$ and the emissivity is equal to the absorption. Temperature is just $power^{1/4}$ and the total power getting through to the final filter is just:

$$\sum_{i=1}^{N+1} (1-a)^{N+1-i} * \frac{P_{N_i}}{P_{N_1}} \tag{6}$$

I am including the Mathematica sheet at the end to clarify the matrix algebra, but I found that to absorb 90% of the incident radiation, we need 17-20 filters. With the spacing given in [3] of 0.25" and a Zitex thickness of 0.015", this would be 5" for 20 filters. It us unclear how changing the spacing will effect transmission. Note that our area is 0.16 m^2, and so the power goes from 75 Watts to 4.97W.

4 Literature Suggestions for Alternate Possibilities

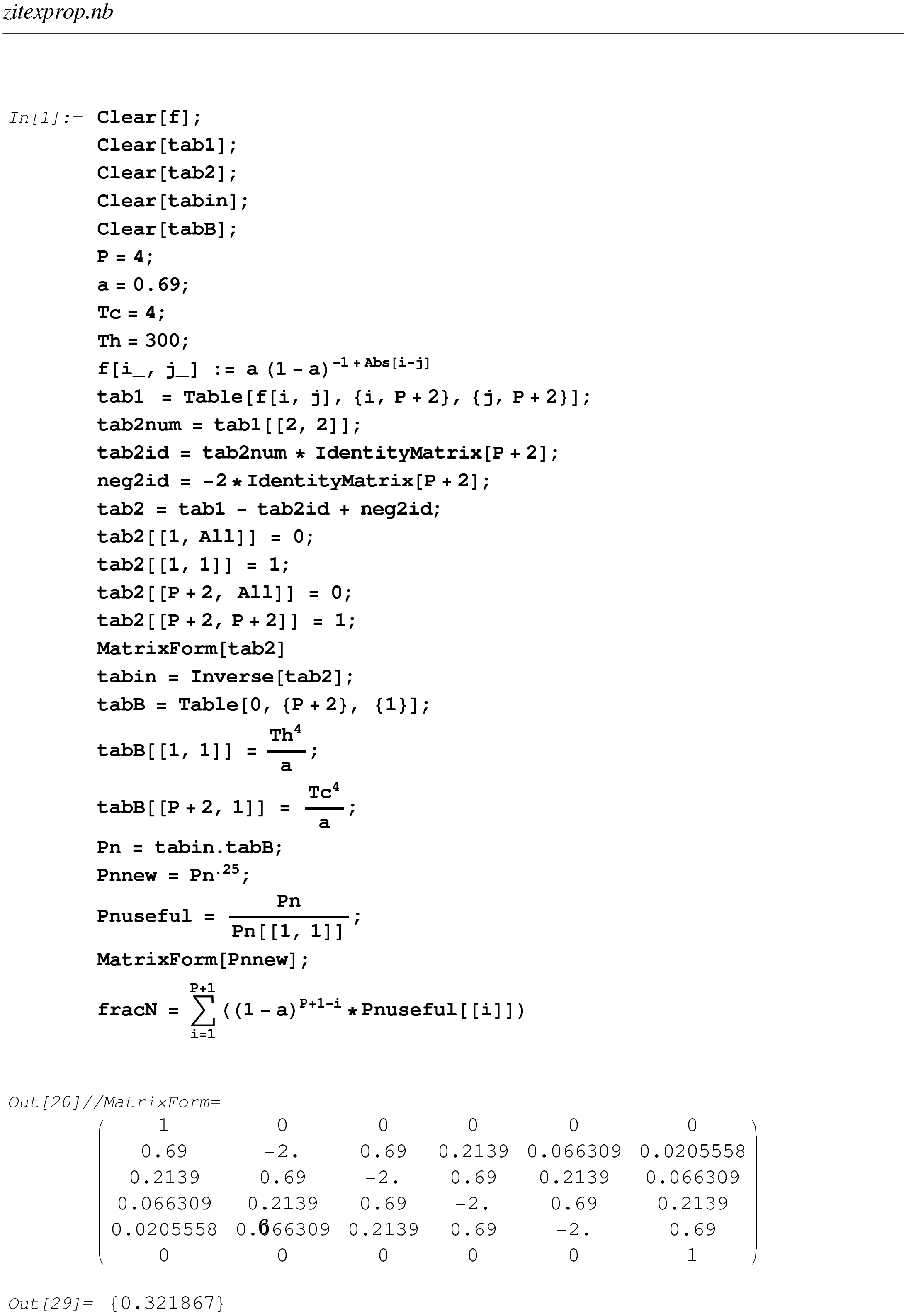
-Dylite, a thicker styrofoam, was included in [3] but not tested for multiple layers.

BIBLIOGRAPHY 1. Brochure faxed to me by St. Gobain. 2. DH Benford, MC Gaidis, JW Kooi, "Optical Properties of Zitex in the Infrared to Submillimeter," Applied Optics, 42, 25, pp 5118-5122. 3. GA Ediss, D Koller, "ALMA Memo No. 412: 68.5 to 118 GHz Measurements of Possible Infrared Filter Materials: Black Polyethylene, Zitex, and Grooved and Un-Grooved Flourogold and HDPE," March 2002. 4. HS Clarke, LR D"Addario, "ALMA Memo No. 269: Tests of Materials for Use in Multi-Layer Infrared Filters in Cryogenic Applications" Appendices

Table 1: N is number of layers, P is the calculated total fraction of power transmitted, T is the calculated temperature of the last layer in Kelvin, and total power is the energy in the last layer

N	% Power Transmitted	T(K)	Total Power (W/ m^2
0	1	300	459
4	0.32	211	112.39
10	0.16	177	55.65
12	0.136	170	47.36
15	0.112	162	39.15
16	0.106	160	37.16
17	0.1	158	35.36
18	0.095	156	33.58
19	0.0908	154	31.89
20	0.0867	152	30.26
208	0.0067	80.33	2.36

Figure 1: Mathematica Sheet of Zitex Calculation: a=0.69 (as measured by [4])



zitexprop.nb

```
In[1]:= Clear[f];
        Clear[tab1];
        Clear[tab2];
        Clear[tabin];
        Clear[tabB];
        P = 4;
        a = 0.69;
        Tc = 4;
        Th = 300;
        f[i_, j_] := a (1 - a)^(-1 + Abs[i-j])
        tab1  = Table[f[i, j], {i, P + 2}, {j, P + 2}];
        tab2num = tab1[[2, 2]];
        tab2id = tab2num * IdentityMatrix[P + 2];
        neg2id = -2 * IdentityMatrix[P + 2];
        tab2 = tab1 - tab2id + neg2id;
        tab2[[1, All]] = 0;
        tab2[[1, 1]] = 1;
        tab2[[P + 2, All]] = 0;
        tab2[[P + 2, P + 2]] = 1;
        MatrixForm[tab2]
        tabin = Inverse[tab2];
        tabB = Table[0, {P + 2}, {1}];
        tabB[[1, 1]] = Th^4/a;
        tabB[[P + 2, 1]] = Tc^4/a;
        Pn = tabin.tabB;
        Pnnew = Pn^.25;
        Pnuseful = Pn/Pn[[1, 1]];
        MatrixForm[Pnnew];
        fracN = Sum[((1 - a)^(P+1-i) * Pnuseful[[i]]), {i, 1, P+1}]
```

```
Out[20]//MatrixForm=
      1           0          0         0          0          0
     0.69        -2.        0.69     0.2139    0.066309   0.0205558
    0.2139       0.69       -2.       0.69      0.2139    0.066309
   0.066309     0.2139      0.69      -2.        0.69      0.2139
   0.0205558   0.066309    0.2139     0.69       -2.        0.69
      0           0          0         0          0          1

Out[29]= {0.321867}
```

Figure 2: Appendix: Mathematica Sheet of Zitex Calculation: a=0.79

```
In[30]:= Clear[f];
         Clear[tab1];
         Clear[tab2];
         Clear[tabin];
         Clear[tabB];
         P = 4;
         a = 0.79;
         Tc = 4;
         Th = 300;
         f[i_, j_] := a (1 - a)^(-1 + Abs[i-j])
         tab1 = Table[f[i, j], {i, P + 2}, {j, P + 2}];
         tab2num = tab1[[2, 2]];
         tab2id = tab2num * IdentityMatrix[P + 2];
         neg2id = -2 * IdentityMatrix[P + 2];
         tab2 = tab1 - tab2id + neg2id;
         tab2[[1, All]] = 0;
         tab2[[1, 1]] = 1;
         tab2[[P + 2, All]] = 0;
         tab2[[P + 2, P + 2]] = 1;
         MatrixForm[tab2]
         tabin = Inverse[tab2];
         tabB = Table[0, {P + 2}, {1}];
                           Th^4
         tabB[[1, 1]] = ────;
                            a
                               Tc^4
         tabB[[P + 2, 1]] = ────;
                                a
         Pn = tabin.tabB;
         Pnnew = Pn^.25;
                           Pn
         Pnuseful = ──────────────;
                      Pn[[1, 1]]
         MatrixForm[Pnnew];
                   P+1
         fracN =   ∑  ((1 - a)^(P+1-i) * Pnuseful[[i]])
                   i=1

Out[49]//MatrixForm=
       ⎛    1           0          0         0         0          0     ⎞
       ⎜  0.79        -2.        0.79     0.1659   0.034839  0.00731619 ⎟
       ⎜  0.1659      0.79       -2.       0.79     0.1659    0.034839  ⎟
       ⎜ 0.034839     0.1659     0.79      -2.       0.79      0.1659   ⎟
       ⎜0.00731619  07034839    0.1659     0.79      -2.        0.79    ⎟
       ⎝    0           0          0         0         0          1     ⎠

Out[58]= {0.276888}
```