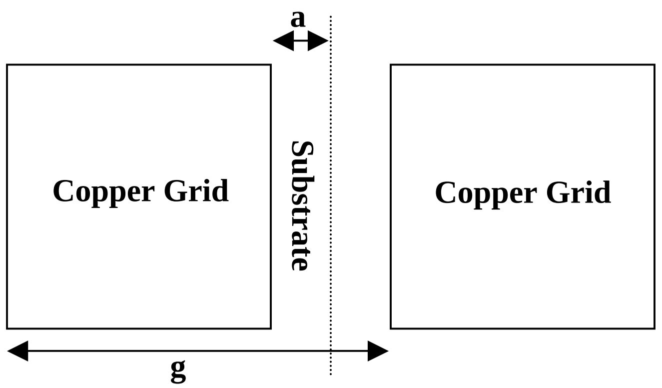
Low-pass Filters

We have researched low-pass capacitive grid filters using copper evaporated onto a dielectric substrate. The grids act as radiation filters for higher frequencies. Inductive grid filters act as radiation filters for lower frequencies. I have outlined the findings to-date, though they will certainly be updated/changed going forward.

The following diagram shows a sample grid with the key variables used throughout this document:



“a” is half of the gap between copper grids. “g” is the periodicity, which is the center to center distance of the grids. The ratio a/g is commonly used and will be useful when we look at the effects of varying “a” and “g” on the properties of the grids.

The capacitive grids exhibit behavior like that of an RLC circuit. Incident radiation arrives at the grid and is either transmitted through or reflected (I assume negligible absorption for now). The size of “a” and “g”, and the ratio “a/g”, determine what frequency ranges are transmitted/reflected. The RLC properties of the grids take the incident radiation and shunt a portion of it, which then gets re-radiated. This causes you to loose some of the transmittance in the range that you wish to pass, but this amount should be insignificant compared to the amount shunted and re-radiated in the stop-band. Capacitive mesh filters, such as this, have resonance at λ=g (typically at 1.0g to 1.1g), at which point the grids are totally reflecting (as a note, inductive meshes are totally transmitting at resonance). The normalized frequency for these grids is w=g/λ which puts the resonant frequency, w0, at or close to 1.

Single Layer Grids

The key formulas used in understanding a single grid transmission in Mathematica are below ("Equivalent-circuit formulas for metal grid reflectors at dielectric boundary" , Lewis Whitbourn and Richard Compton, Applied Optics Vol. 24 1984):

Mathematica:

$$\mathbf{Reactance[f\_] := \frac{2}{n1^2 + n2^2} \left(1 \Big/ \left(4\, w\, Log\left[Csc\left[\frac{\pi\, a}{g}\right]\right]\right)\right) \left(\frac{2\, \pi\, g\, f}{c} - \frac{c}{2\, \pi\, g\, f}\right)}$$

$$\mathbf{Resistance[f\_] := \sqrt{\frac{\pi\, eps\, f}{bulk}}\ \frac{1}{1 - \frac{2\, a}{g}}}$$

$$\mathbf{w := w0 \sqrt{\frac{2}{n1^2 + n2^2}}}$$

$$\mathbf{n1 := 1}$$

$$\mathbf{w0 := 1}$$

$$\mathbf{n2 := 1.7}$$

$$\mathbf{g := .00002}$$

$$\mathbf{a := .000002}$$

$$\mathbf{c := 3 * 10^{8}}$$

$$\mathbf{eps := 8.85 * 10^{-12}}$$

$$\mathbf{bulk := 5.8 * 10^{7}}$$

$$\mathbf{Transmission[f\_] := \left(4\, n1\, n2 \left(Resistance[f]^2 + Reactance[f]^2\right)\right) / \left((1 + (n1 + n2)\ Resistance[f])^2 + (n2 + n1)^2\ Reactance[f]^2\right)}$$

$$\mathbf{Reflection[f\_] := \left((1 + (n2 - n1)\ Resistance[f])^2 + (n2 - n1)^2\ Reactance[f]^2\right) / \left((1 + (n2 + n1)\ Resistance[f])^2 + (n2 + n1)^2\ Reactance[f]^2\right)}$$

$$\mathbf{Absorption[f\_] := (4\, n1\ Resistance[f]) / \left((1 + (n2 + n1)\ Resistance[f])^2 + (n2 + n1)^2\ Reactance[f]^2\right)}$$

A more simplified version of the equations above that can be used in Mathematica for a single layer filter is below ("Effective low-pass filters for far infrared frequencies", R. Ulrich, Infrared Physics Vol. 7 1967):

Mathematica:

$$Z := \frac{1}{\mathrm{Log}\left[\mathrm{Csc}\left[\frac{\pi a}{2g}\right]\right]}$$

$$\mathrm{Trans}[f\_] := \frac{R^2 + Z^2 \left(\frac{2\pi g f}{c} - \frac{c}{2\pi g f}\right)^2}{(1 + R)^2 + Z^2 \left(\frac{2\pi g f}{c} - \frac{c}{2\pi g f}\right)^2}$$

$$g := .000014$$

$$a := .000002$$

$$c := 3 * 10^8$$

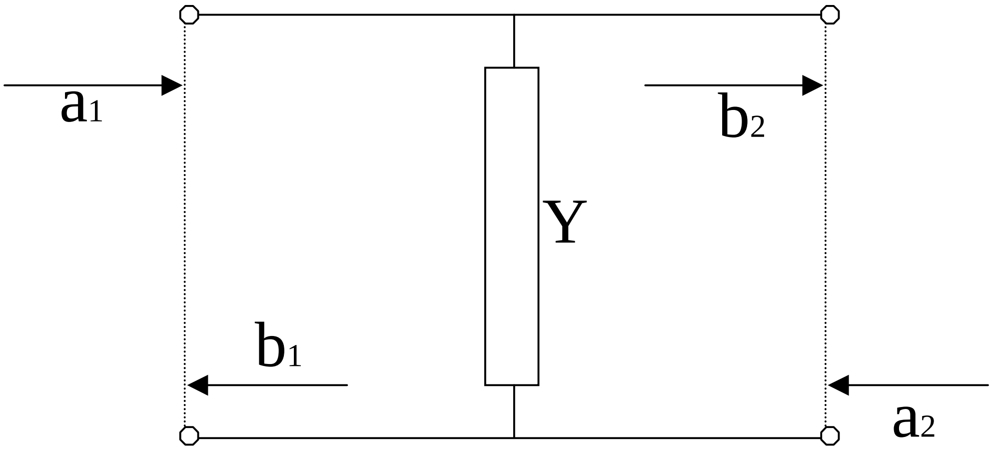
$$R := .01$$

The equations from Ulrich are good for a/g less than or equal to .12. Also, the loss-resistance R is the absorption of the capacitive grid, which Ulrich says is practically on the order of .01. Ulrich suggests that if you use multiple layers then a guideline for the distance "d" between layers is d = g/2w0 where w0 is the natural angular frequency of the grid. In addition, if the grids are spced beyond where interference plays a significant role, then the transmittivities of each grid can be multiplied to get the overall power transmission.

The two sets of equations above agree very well in Mathematica, and agree well with HFSS simulations, suggesting their validity.

Multiple Layer Grids
For multiple layers, a new set of equations must be used and a cascading matrix developed. The general concept is that at each interface, a matrix can be used to represent the transmittance/reflectance. The matrices can later be multiplied to arrive at the overall transmittance of the filter.

The following two-port diagram helps explain the matrix operations:



a1 and a2 represent the incident wave amplitudes and b1 and b2 represent the reflected wave amplitudes. Y represents the shunt admittance in the "circuit", which effectively acts to shunt incident radiation, which then gets re-radiated. The following relationships were found to hold true:

Incident radiation: $a1 = r21*a2 + r22*b2$
Reflected radiation: $b1 = r11*a2 + r12*b2$
Resonant frequency: $w0 = 2\pi/g$
Normalized impedance Of L and C at resonance: $Z0 = w0*L = 1/(w0*c)$
Generalized frequency: $\Omega = w/w0 - w0/w$
Normalized admittance: $Y = 1/(R + i*Z0*\Omega)$
Grid admittance: $Y0 = 1/Z0 = 2(n1\hat{}2 + n2\hat{}2)*\ln[\csc(\pi a/g)]$

The coefficients rij define the cascading matrix for the metallic grid layer. The reflectivity for a wave coming from the left is b1/a1 = r12/r22. The amplitude transmittance is 1/r22. The paper terms this the matrix for the shunt admittance (Y) at the junction of two transmission lines, which makes sense in that this represents the metallic grid which acts to shunt radiation through its RLC properties. The matrix for rij is below.

$$r(Y) = 1/2n1 \begin{vmatrix} -Y + (n1 + n2) & -Y + (n1 - n2) \\ Y + (n1 - n2) & Y + (n1 + n2) \end{vmatrix}$$

The next layer is the dielectric layer, which they term the transmission line. This has its own matrix, which really is a matrix that accounts for the phase change as the radiation passes through the dielectric substrate of depth "d". The matrix is below:

$$r(d) = \begin{vmatrix} \exp(-i\gamma) & 0 \\ 0 & \exp(i\gamma) \end{vmatrix}$$

where $\gamma = 2\pi nd/\lambda$
λ = vacuum wavelength of the wave

In the event that the dielectric medium is absorbing, you need to add a negative imaginary part to γ (I have ignored this in my calculations).

Finally, if you have a dielectric layer with no metallic grid, you represent this by the r(Y) matrix with Y = 0.

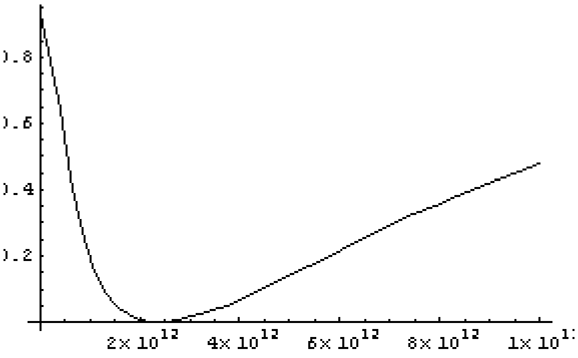
$$r(b) = 1/2n1 \begin{vmatrix} (n1 + n2) & (n1 - n2) \\ (n1 - n2) & (n1 + n2) \end{vmatrix}$$

These three matrices are multiplied together and the process is repeated for each subsequent layer. The result should be a 2x2 matrix that is dependent on f (frequency).

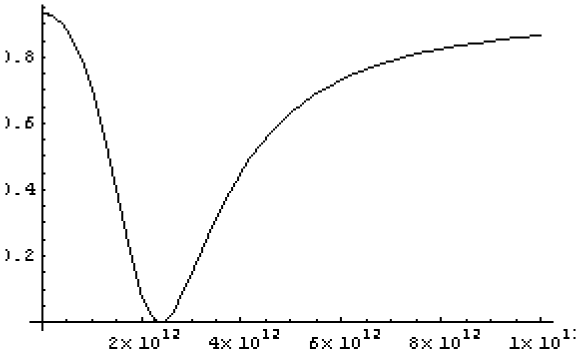
Discoveries to date:
The grids are very dependent on “a”, “g” and “a/g”. It seems that the at or about a ratio of .12 for a/g is where you get a steep and narrow stop-band. Generally, the following rules hold true for a single grid:

1. Keeping “g” constant and changing “a” will not change the middle of your stop-band. It has the effect of narrowing the stop band. It also has the effect of making the slope from the transmission range to the stop-band range less steep. An example is below:

a: 1 micron g: 20 microns a/g: .05

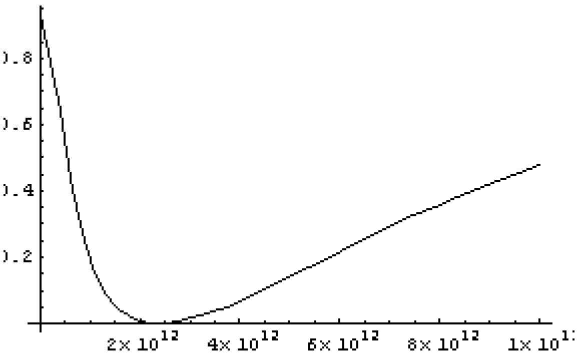


a: 4 microns g: 20 microns a/g: .20

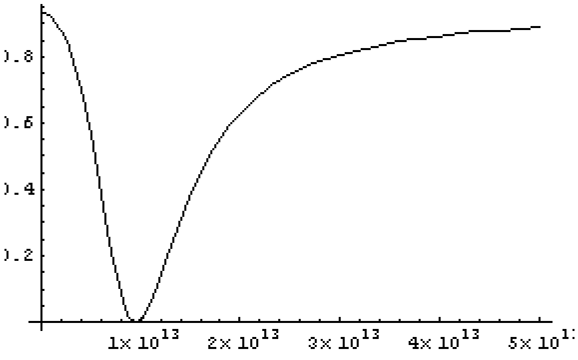


2. Keeping “a” constant and changing “g” has the effect of shifting the middle of the stop-band. As “g” decreases, the stop-band shifts to higher frequencies. Decreasing “g” also has the effect of making the slope from the transmission range to the stop-band range less steep. It has the effect of narrowing the stop band. Examples are below:

a: 1 micron g: 20 microns a/g: .05

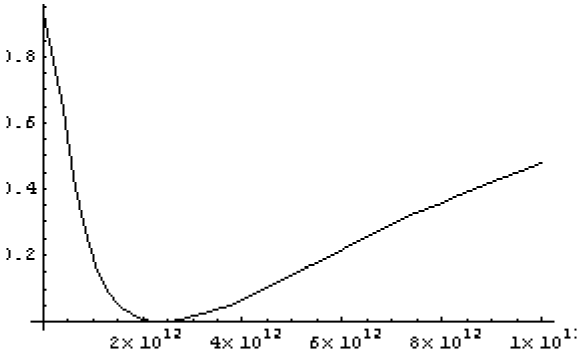


a: 1 micron g: 5 microns a/g: .20

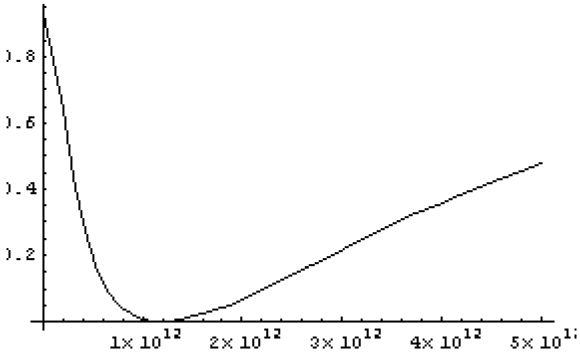


3. Keeping the same "a/g" ratio but varying the size of "a" and "g" (i.e., scaling the grids with the same proportionality), changes the grids in the following way. If you increase the grid size you shift the stop-band to lower frequencies. If you decrease the grid size then you shift the stop-band to higher frequencies. The shift is not linear, however, as you will see below. The broadness of the stop-band does not seem to get impacted in any way.

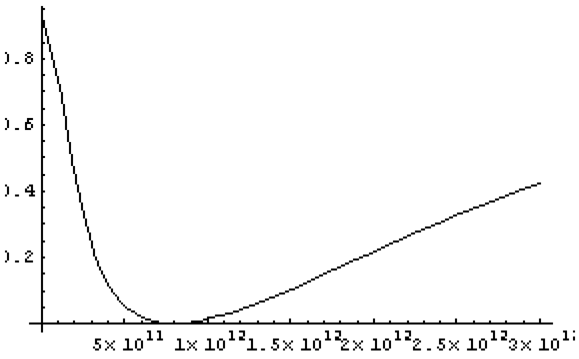
a: 1 micron g: 20 microns a/g: .05



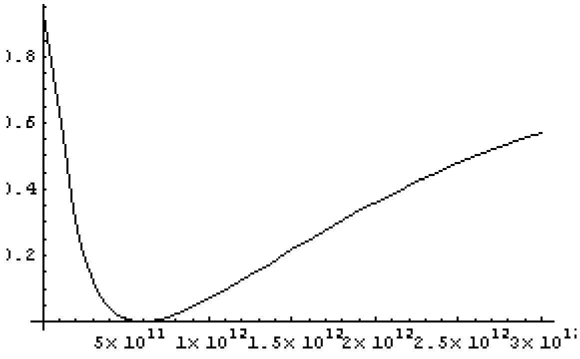
a: 2 micron g: 40 microns a/g: .05



a: 3 micron g: 60 microns a/g: .05



a: 4 micron g: 80 microns a/g: .05



Below is a table with the suggested dimensions of grids to block certain frequencies. The grid assumes an a/g ratio of .10. Multiple grids are necessary to block larger ranges but they can easily be assembled using grids form this table.

a (microns)	g (microns)	a/g	Stop-band (GHz)
30	300	0.1	150
10	100	0.1	500
5	50	0.1	950
4	40	0.1	1100
3	30	0.1	1700
2	20	0.1	2600
1	10	0.1	5000